**Homework #6 (10 points) - Show all work on the following problems:**

**Problem 1 (3 points):** This problem looks at the properties of EM waves in conductors.

**1a (1 point):** Show that the skin depth $1/k_i$ in a poor conductor ($\sigma \ll \omega\varepsilon$) is $\frac{2}{\sigma}\sqrt{\varepsilon/\mu}$ regardless of frequency.

**1b (1 point):** Show that the skin depth $1/k_i$ in a good conductor ($\sigma \gg \omega\varepsilon$) is $\lambda/(2\pi)$.

**1c (1 point):** Show that in a good conductor the magnetic field lags the electric field by 45°.

**Problem 2 (3 points):** Start with the real form for EM plane waves in a conductor:

$$\vec{E}(z,t) = E_0 e^{-k_i z}\cos(k_r z - \omega t + \delta)\,\hat{x}$$
$$\vec{B}(z,t) = B_0 e^{-k_i z}\cos(k_r z - \omega t + \delta + \phi)\,\hat{y}$$

**2a (2 points):** Show that the time-averaged energy density is $\langle u\rangle = \frac{k_r{}^2}{2\mu\omega^2}E_0{}^2 e^{-2k_i z}$, and show that the magnetic portion of the energy always dominates (unlike in vacuum or in a dielectric, where they are equal).

**2b (1 point):** Show that the intensity (time-averaged energy flux) of the light in the conductor is $I = \frac{k_r}{2\mu\omega}E_0{}^2 e^{-2k_i z}$.

**Problem 3 (4 points):** Show that the energy in a Transverse Electric (TE) waveguide propagates at the group velocity. Start with the real solutions for the fields (note $E_z = 0$).

$$B_x = \frac{kB_0}{(\omega/c)^2 - k^2}\frac{m\pi}{a}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin(kz - \omega t + \delta)$$
$$B_y = \frac{kB_0}{(\omega/c)^2 - k^2}\frac{n\pi}{b}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin(kz - \omega t + \delta)$$
$$B_z = B_0\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos(kz - \omega t + \delta)$$
$$E_x = \frac{\omega B_0}{(\omega/c)^2 - k^2}\frac{n\pi}{b}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin(kz - \omega t + \delta)$$
$$E_y = \frac{-\omega B_0}{(\omega/c)^2 - k^2}\frac{m\pi}{a}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin(kz - \omega t + \delta)$$

**3a (2 points):** Find the average energy per unit time $\int\langle\vec{S}\rangle\cdot\overrightarrow{da}$ traveling through the waveguide, by integrating the intensity over the cross section of the waveguide.

**3b (2 points):** Find the average energy per unit length $\int\langle u\rangle\cdot da$ contained in the waveguide, and show that the ratio between the two quantities is the group velocity $v_g$. *Hint: In the later steps, express things in terms of the cutoff frequency $\omega_{mn}$ to simplify.*